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APPLICATION OF THE METHOD OF PARTITIONING THE STATE OF STRESS TO THE ANALYSIS OF THERMOELASTIC SHELLS

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By using an asymptotic approach [1], the method of partitioning the state of stress is extended to thermoelastic shells. It is examined in detail in [2] for unheated shells subjected to the effect of external forces, and consists of representing the total state of stress of the shell as the sum of those simpler states of stress for each of which the simplest methods for their construction can be given.

Partitioning of the state of stress was performed in [3] for shells with a constant temperature over the thickness. It was noted in [4] in an analysis of a circular cylindrical shell by bending theory that integrals extended over the whole middle surface, which describe the fundamental state of stress, and integrals which damp out with distance from the edges and represent edge effects are contained in the general solution. In a number of papers, [5] for example, partitioning is performed on the basis of graphic physical representations for simple examples of analyzing circular cylindrical shells.

A general approach to the analysis of rigid thermoelastic shells by the partitioning method is described below.

1. We agree to understand rigid shells to be those with such support of the edges which will eliminate pure bending strains (the membrane state of stress predominates for such supports in an unheated shell upon compliance with all the other conditions for applicability of membrane theory far from the edges).

Henceforth we shall assume that the temperature varies arbitrarily over the shell thickness. The notation used and the equations are taken from [2].

Let us proceed from the general equations of shell theory, compiled under the assumption that the middle surface is referred to an orthogonal disconnected coordinate system α_1, α_2.

The equilibrium equations have the form (we consider there to be no surface load and the state of stress and strain to be caused by the temperature field)

$$\frac{1}{A_i}\frac{1}{R}\frac{\partial T_{i*}}{\partial \xi_i} + \frac{1}{A_j}\frac{1}{R}\frac{\partial S_{ij*}}{\partial \xi_j} + h_*^s k_j (T_{i*} - T_{j*}) + \tag{1.1}$$

$$h_*^s k_i (S_{ij*} + S_{ji*}) - h_*^{2s}\frac{N_{i*}}{R_i'} + h_*^{2s}\frac{N_{j*}}{R_{12}} = 0$$

$$\frac{T_{1*}}{R_1'} + \frac{T_{2*}}{R_2'} - \frac{S_{12*} + S_{21*}}{R_{12}} + \frac{1}{A_i}\frac{1}{R}\frac{\partial N_{i*}}{\partial \xi_i} +$$

$$\frac{1}{A_j}\frac{1}{R}\frac{\partial N_{j*}}{\partial \xi_j} + h_*^s k_j N_{i*} + h_*^s k_i N_{j*} = 0$$

$$\frac{1}{A_i}\frac{1}{R}\frac{\partial G_{i*}}{\partial \xi_i} - \frac{h_*^{2-4s}}{A_j}\frac{1}{R}\frac{\partial H_{ij*}}{\partial \xi_j} + h_*^s k_j (G_{i*} - G_{j*}) - \tag{1.2}$$
$$h_*^{2-3s} k_j (H_{ij*} + H_{ji*}) - N_{i*} = 0$$

The elasticity relationships are

$$T_{i*} = \frac{2Eh}{(1-\nu^2)}\frac{1}{R}(\varepsilon_{i*} + \nu\varepsilon_{j*}) - t_{0*}, \quad S_* = S_{ij*} = S_{ji*} = \frac{Eh}{(1+\nu)}\frac{1}{R}\,\omega_* \tag{1.3}$$

$$G_{i*} = -h_*^{2-4s}\frac{2Eh}{3(1-\nu^2)}(\varkappa_{i*} + \nu\varkappa_{j*}) + t_* \tag{1.4}$$
$$H_* = H_{ij*} = H_{ji*} = \frac{2Eh}{3(1+\nu)}\tau_*$$

The strain-displacement formulas are

$$\varepsilon_{i*} = \frac{1}{A_i}\frac{\partial u_{i*}}{\partial \xi_i} + h_*^s k_i R u_{j*} - \frac{R}{R_i'} w_* \tag{1.5}$$

$$\omega_{i*} = \frac{1}{A_i}\frac{\partial u_{j*}}{\partial \xi_i} - h_*^s k_i R u_{i*} + \frac{R}{R_{12}} w_*, \quad \omega_* = \omega_{1*} + \omega_{2*}$$

$$\gamma_{i*} = -\frac{1}{A_i}\frac{\partial w_*}{\partial \xi_i} - h_*^{2s}\frac{R}{R_i'} u_{i*} - h_*^{2s}\frac{R}{R_{12}} u_{j*} \tag{1.6}$$

$$\varkappa_{i*} = -\frac{1}{A_i}\frac{\partial \gamma_{i*}}{\partial \xi_i} - h_*^s k_i R \gamma_{j*}$$

$$\tau_* = -\frac{1}{A_1}\frac{\partial \gamma_{2*}}{\partial \xi_1} + h_*^s k_1 R \gamma_{1*} - h_*^{2s}\frac{R}{R_{12}}\varepsilon_{2*}$$

$$\left(k_i = \frac{1}{A_i A_j}\frac{\partial A_i}{\partial \alpha_j}\right)$$

For convenience in the subsequent exposition, asterisks have been placed to the right of the desired quantities in (1.1) — (1.6) and some new notation has been introduced: ξ_i, R, h_*. As yet they must be replaced by α_i, 1, 1, respectively, and the asterisks must be omitted.

As usual, the integral characteristics are introduced in place of the temperature T in the elasticity relationships (γ is the normal coordinate to the middle surface)

$$t_0 = \frac{E\alpha_t}{1-\nu}\int_{-h}^{+h} T\,d\gamma, \quad t = \frac{E\alpha_t}{1-\nu}\int_{-h}^{+h} T\gamma\,d\gamma \tag{1.7}$$

The formula for passing from the forces and moments to the stresses under an arbitrary law of temperature variation over the thickness has the form (see [4, 6], for example):

$$\tau_i = \frac{T_i}{2h} - \gamma\frac{3G_i}{2h^3} + \frac{t_0}{2h} - \gamma\frac{3t}{2h^3} - \frac{E\alpha_t}{1-\nu}T \tag{1.8}$$

In the case of a linear temperature over the thickness, only the first two members in the right side remain in (1.8).

Here and henceforth, each equality containing the subscripts i and j should be considered as two: one is obtained for $i = 1$, $j = 2$, and the other for $i = 2$, $j = 1$.

2. Using the linearity of the problem, let us examine separately the states of stress corresponding to the temperature effects t_0 and t in the notation (1.7). The case ($t_0 \neq$

$0,\ t = 0)$ is investigated in [3] where it is shown that the membrane state of stress holds in rigid shells with a constant temperature over the thickness.

Let us analyze shells with a temperature field $(t_0 = 0,\ t \neq 0)$.

The asymptotics of the desired quantities of the state of stress and strain of a thermoelastic shell has the following form:

$$t = h_*{}^{\circ} t_*,\ 2Ehu_i = h_*{}^{-s} 2Ehu_{i*},\ 2Ehw = h^{-2s}\ 2Ehw_* \tag{2.1}$$
$$(T_i,\ S_{ij}) = h_*{}^{-2s}\ (T_{i*},\ S_{ij*}),\quad N_i = h_*{}^{-s} N_{i*}$$
$$G_i = h_*{}^{\circ}\ G_{i*},\quad H_{ij} = h_*{}^{2-4s} H_{ij*}$$

The quantities $t_*,\ 2Ehu_{i*},\ \ldots,\ H_{ij*}$ in (2.1) are of the same order, h_* is the relative half-thickness of the shell, and s is the exponent of variability of the fundamental state of stress.

The asymptotics (2.1) is selected by using reasoning analogous to that made in [1]. The powers of h_* there are selected in such a way that the boundary value problems which are obtained for the asymptotics taken as $h_* \to 0$ would not be contradictory.

We introduce new variables ξ_i in place of α_i by selecting them in such a way that differentiation with respect to them would not result in a substantial increase in the functions required

$$\alpha_i = h_*{}^{s}\ R\xi_i \tag{2.2}$$

Substituting (2.1), (2.2) into the shell theory equations, we consequently obtain the system of equations (1.1) — (1.6). Discarding terms with the small factors $h_*{}^{2-4s}$ and $h_*{}^{2-3s}$ in (1.2) and (1.4), and returning to the previous notation, we obtain

$$G_i = t,\quad N_i = \frac{1}{A_i}\frac{\partial G_i}{\partial \alpha_i} = \frac{1}{A_i}\frac{\partial t}{\partial \alpha_i} \tag{2.3}$$

Together with the equilibrium equations (1.1) the elasticity relationships (1.3) and the tangential strain-displacement formulas (1.5), the formulas (2.3) form a complete system of equations. Since the bending moments and transverse forces are known, then determination of the displacements and tangential forces reduces to integrating a system of equations which formally agrees with the membrane equations. The terms $O\ (h_*{}^{2-4s})$ and $O\ (h_*{}^{2-3s})$ are discarded in the construction of this theory, hence, the error ε of the system obtained is estimated as:

$$\varepsilon = O\ (h_*{}^{2-4s}) \tag{2.4}$$

Let us note that the role of the quantities t_0 and t in the state of stress and strain of a rigid shell can be estimated. An asymptotic analysis of the thermoelasticity equations shows that if the quantity t_0 is commensurate with unity and t with $h_*{}^{c}$, then for $c = 2s$ the state of stress and strain caused separately by the temperature effects t_0 and t are commensurate with the displacements, and for $c = 2-2s$ with the stresses. If $c > 2-2s$ the quantity t can be neglected to the accuracy of quantities $O\ (h_*^{2-4s} + h_*^{c-2+2s})$ in the analysis of a shell.

3. As mentioned above, a membrane state of stress occurs in rigid thermoelastic shells for $t_0 \neq 0,\ t = 0$. In this case, the question of separation of the boundary conditions into conditions for the membrane equations and the condition for the simple edge effect is solved exactly as in the nontemperature problem: the tangential conditions are satisfied because of the arbitrariness of the membrane system of equations, and the residuals which appear in the nontangential conditions are removed by using the simple edge effect. Consequently, secondary residuals appear in the tangential conditions, which turn out to be small. It can be seen in the examples of simply- and hinge-supported edges that the smallness of the secondary residuals is assured by the fact that the moments and transverse forces are considerably smaller than the tangential forces in the membrane state of stress.

Let $t_0 = 0$, $t \neq 0$. Then, as has been shown in Sect. 2, the system of equations (1.1), (1.3), (1.5), (2.3), which agrees formally with the membrane equations, must be integrated in order to determine the desired quantities in the thermoelastic problem.

The boundary conditions in this case must also be selected so that the secondary residuals which appear as a result of an analysis by the partitioning method, would be small. A difficulty hence arises which is related to the fact that the bending moments and transverse forces for $t_0 = 0$ and $t \neq 0$ are commensurate with the tangential forces. An analogous situation, which occurs in an unheated shell with all edges simply supported, is considered in the monograph [2]. Let us use the method applied in [2] to derive those boundary conditions which should be taken into account in determining the fundamental state of stress and the simple edge effect in a rigid thermoelastic shell with $t_0 = 0$, $t \neq 0$.

Henceforth, we shall need the asymptotics of quantities of the simple edge effect. Let us write it by using [7] and considering that the edge to be given by the equation $\alpha_1 = \alpha_{10}$

$$2Ehw = h_*^0\, 2Ehw_*, \quad 2Ehu_i = h_*^{1/2}\, 2Ehu_{i*} \tag{3.1}$$

$$S_{ij} = h_*^{1/2-s} S_{ij*}, \quad 2Eh\gamma_1 = h_*^{-1/2} 2Eh\gamma_{1*}, \quad T_1 = h_*^{1/2} T_{1*}$$

$$T_2 = h_*^0 T_{2*}, \quad H_{ij} = h_*^{3/2-s} H_{ij*}, \quad N_1 = h_*^{1/2} N_{1*}, \quad G_i = h_*^1 G_{i*}$$

The powers of the small parameter h_* in (3.1) are selected in such a way that the quantities $2Ehw_*$, $2Ehu_{i*}, \ldots, G_{i*}$ would be of the same order.

Let M denote any of the desired quantities of the state of stress. Following [7], let us give all the required quantities in the form of the sum of three terms

$$M = M^{(p)} + h_*^b M^{(h)} + h_*^a M^{(e)} \tag{3.2}$$

in which $M^{(p)}$ is the particular solution of the membrane equations, $M^{(h)}$ is the solution of the homogeneous membrane equations, and $M^{(e)}$ is the solution of the simple edge effect equations. The quantities $M^{(h)}$ and $M^{(e)}$ are found from the homogeneous equations, hence, they are preceded by the scale factors h_*^a and h_*^b which will be selected as a function of the boundary conditions. The numbers a and b characterize the intensity of the state of stress corresponding to the homogeneous solution of the membrane equations and the simple edge effect equations.

Let the shell have the simply supported edge $\alpha_1 = \alpha_{10}$ (it is assumed that there is still a clamped edge which makes the shell rigid). In this case the boundary conditions are the following:

$$T_1 = 0, \quad S_{12} = 0, \ G_1 = 0, \ N_1 = 0$$

Within the framework of the accuracy taken, the corrections due to the torsional moments can be neglected in imposing the boundary conditions.

Taking account of (3.1), (3.2), (2.1), (2.3), let us represent the boundary conditions in the form

$$h_*^{-2s}\,(T_{1*}^{(p)} + h_*{}^b T_{1*}^{(h)}) + h_*^{a+1/2} T_{1*}^{(e)} = 0 \qquad (3.3)$$
$$h_*^{-2s}\,(S_{12*}^{(p)} + h_*{}^b S_{12*}^{(h)}) + h_*^{a+1/2-s} S_{12*}^{(e)} = 0$$
$$h_*{}^0 t_* + h_*^{a+1} G_{1*}^{(e)} = 0, \quad h_*^{-s}\,\frac{1}{A_1}\,\frac{1}{R}\,\frac{\partial t_*}{\partial \xi_1} + h_*^{a+1/2} N_{1*}^{(e)} = 0$$

Here and later s is the exponent of variability of the fundamental state of stress. As has been shown in [1], the values of s should satisfy the inequality $0 \leqslant s < 1/2$.

Let us select a and b so as to obtain noncontradictory boundary value problems for the fundamental state of stress and the edge effect. We set $a = -1$. Retaining only the main terms in the last two formulas of (3.3), we obtain the following conditions for the edge effect in a quite rough approximation:

$$G_1{}^{(e)} = -t, \quad N_1{}^{(e)} = 0 \qquad (3.4)$$

The conditions for the fundamental state of stress are given by the first two formulas in (3.3). We express the quantities $T_1{}^{(e)}$ and $S_{12}{}^{(e)}$ in terms of t. To this end, we use formulas for the simple edge effect quantities derived in [7], where an iteration process has been constructed which permits determination of the edge effect quantities to any accuracy. Following [7], we represent each of the edge effect quantities as an expansion

$$P_*^{(e)} = \sum_{i=0}^{\infty} h_*^{i(1/2-s)} P_{,i}^{(e)} \qquad (3.5)$$

The number i after the comma denotes the number of the approximation.

In order to determine $T_{1,0}^{(e)}$ and $S_{12,0}^{(e)}$ on the edge, we take the zero approximation formulas in [6] and satisfy the boundary conditions (3.4), we consequently find

$$T_{1,0}^{(e)} = S_{12,0}^{(e)} = 0 \qquad (3.6)$$

Let us insert the expansion of the quantities $T_1{}^{(e)}$, $S_{12}{}^{(e)}$ of the form (3.5) into (3.3), take account of (3.6) and set $b = 0$, then by discarding terms $O\,(h_*^{1/2-s})$ in the formulas, we obtain

$$T_{1*}^{(p)} + T_{1*}^{(h)} + T_{1,1}^{(e)} = 0, \quad S_{12*}^{(p)} + S_{12*}^{(h)} + S_{12,1}^{(e)} = 0 \qquad (3.7)$$
$$N_{1,1}^{(e)} = -\frac{1}{A_1}\,\frac{\partial t_*}{\partial \xi_1}, \quad G_{1,1}^{(e)} = 0 \qquad (3.8)$$

The boundary values $T_{1,1}^{(e)}$ and $S_{12,1}^{(e)}$ can be found by using the first approximation formulas [7] and taking account of conditions (3.8). Without reproducing the computations, we write the final result, the boundary conditions for the fundamental state of stress

$$T_1^{(p)} + T_1^{(h)} = -A_1\left[\frac{1}{A_2}\,\frac{\partial}{\partial \alpha_2} + 2\left(-\frac{k_2 R_2'}{R_{12}} + k_1\right)\right]\frac{1}{A_2}\,\frac{\partial}{\partial \alpha_2}\,\frac{R_2'}{A_1}\,t + \qquad (3.9)$$
$$\frac{1}{R_1'}\,(k_2{}^2 R_2' + 1)\,t - k_2 R_2'\,\frac{1}{A_1}\,\frac{\partial t}{\partial \alpha_1}$$
$$S_{12}^{(p)} + S_{12}^{(h)} = \left[\frac{1}{A_2}\,\frac{\partial}{\partial \alpha_2}\,R_2' k_2\left(1 - \frac{R_2'}{R_1'}\right) - k_1 k_2 R_2' - \frac{1}{R_{12}}\right]t -$$
$$2\left(\frac{1}{A_2}\,\frac{\partial}{\partial \alpha_2}\,\frac{R_2'}{R_{12}} + k_2\right)\frac{A_1}{A_2}\,\frac{\partial}{\partial \alpha_2}\,\frac{R_2'}{A_1}\,t + \frac{1}{A_2}\,\frac{\partial}{\partial \alpha_2}\,R_2'\,\frac{1}{A_1}\,\frac{\partial}{\partial \alpha_1}$$
$$(\alpha_1 = \alpha_{10})$$

Let us consider a hinge-supported edge, on which the boundary conditions are written as follows:

$$T_1 = 0, \; u_2 = 0, \; G_1 = 0, \; w = 0 \quad (\alpha_1 = \alpha_{10})$$

or

$$h_*^{-2s}(T_{1*}^{(p)} + h_*^{b}T_{1*}^{(h)}) + h_*^{a+1/2}T_{1*}^{(e)} = 0$$
$$h_*^{-s}(2Ehu_{2*}^{(p)} + h_*^{b}\,2Ehu_{2*}^{(h)}) + h_*^{a+1/2}2Ehu_{2*}^{(e)} = 0$$
$$h_*^{0}t + h_*^{a+1}G_{1*}^{(e)} = 0$$
$$h_*^{-2s}(2Ehw_*^{(p)} + h_*^{b}2Ehw_*^{(h)}) + h_*^{a}2Ehw_*^{(e)} = 0$$

We set $a = -1$, $b = -1/2 + s$. Discarding small terms in the boundary conditions, we obtain

$$T_{1*}^{(h)} = -h_*^{s}T_{1*}^{(e)}, \quad u_{2*}^{(h)} = -u_{2*}^{(e)} \tag{3.10}$$

$$G_1^{(e)} = -t, \quad w^{(e)} = 0 \tag{3.11}$$

Proceeding exactly as in the case of the simply supported edge, we express $T_{1*}^{(e)}$ and $u_{2*}^{(e)}$ in terms of t. Consequently the boundary conditions for the fundamental state of stress are written as

$$T_1^{(h)} = \mp\frac{\sqrt[4]{3(1-\nu^2)}}{\sqrt{2R_2'h}}k_2R_2't - \frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\frac{1}{A_2}\frac{\partial}{\partial\alpha_2}\frac{R_2'}{A_1}t$$
$$u_2^{(h)} = \mp\frac{R_2'^2\sqrt[4]{3(1-\nu^2)}}{R_{12}\sqrt{2R_2'h}}$$

Here the minus must be selected on the edge for which the inequality $\alpha_1 \geqslant \alpha_{10}$ is satisfied, and the plus for the edge with $\alpha_1 \leqslant \alpha_{10}$.

Remark. *If the quantities k_2 and $1/R_{12}$ vanish simultaneously on the shell edge* (this will occur, for instance, on the edge coincident with the directrix in a cylindrical shell), then the asymptotics of the quantities $T_1^{(e)}$ and $u_2^{(e)}$ varies as follows:

$$T_1^{(e)} = h_*^{1-2s}T_{1*}^{(e)}, \quad u_2^{(e)} = h_*^{1-s}u_{2*}^{(e)}$$

Here b should be selected zero. Then, expressing the edge effect quantities in the conditions for the fundamental state of stress in terms of t, we obtain

$$T_1^{(p)} + T_1^{(h)} = -\frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\frac{1}{A_2}\frac{\partial}{\partial\alpha_2}\frac{R_2'}{A_1}t - 2k_1\frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\frac{R_2'}{A_1}t + \frac{1}{R_1'}t$$
$$u_2^{(p)} + u_2^{(h)} = \frac{1+\nu}{Eh}\frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\frac{R_2'}{A_1}t - \frac{1}{2Eh}\frac{A_1^2}{A_2}\frac{\partial}{\partial\alpha_2}\frac{R_2'^2}{A_1^2}\left(\frac{1}{R_1'} + \frac{\nu}{R_2'}\right)t$$

Let us note that the problem for the simple edge effect is solved independently in the case of the simply- and hinge-supported edges. After it has been solved, we obtain boundary conditions for the fundamental state of stress.

Setting $a = -2s$, $b = 0$, we see that the boundary conditions on the rigidly clamped edge are partitioned as usual

$$u_1^{(p)} + u_1^{(h)} = 0, \quad u_2^{(p)} + u_2^{(h)} = 0$$
$$w^{(e)} = -(w^{(p)} + w^{(h)}), \quad \gamma_1^{(e)} = 0, \quad (\alpha_1 = \alpha_{10})$$

Thus, partitioning has been achieved: the equations of the fundamental state of stress have been obtained, the boundary conditions have been found for the fundamental state of stress and the simple edge effect.

Formulas for the simple edge effect quantities which have been derived to $h_*^{1/2-s}$ accuracy were used in deriving the boundary conditions for the fundamental state of stress. The error (2.4) has been admitted in constructing the equations of the fundamental state of stress of thermoelastic shells. And since the total error is determined by the greatest of those admitted, then the formula

$$\varepsilon = O\left(h_*^{1/2-s}\right) \tag{3.12}$$

holds for the total error of the theory constructed.

4. In order to determine the stresses in the case $(t_0 \neq 0,\ t = 0)$, the forces and moments must be found by integrating the membrane equations and the simple edge effect equations, and then (1.8) should be used. As is shown below, for $t_0 = 0,\ t \neq 0$ the principal stresses on the edge and far from the edge can be calculated without solving the boundary value problem for the membrane state of stress.

We derive formulas for the stresses to the accuracy of (3.12).

Let us examine a simply supported edge. Taking account of (2.1), (3.1), (3.2) and the values of a and b we obtain for the stress τ_2 on the edge

$$\tau_2 = \frac{h_*^{-2s}\left(T_{2*}^{(p)} + T_{2*}^{(h)}\right) + h_*^{-1}T_{2*}^{(e)}}{2h} - \frac{3\gamma}{2h^3}G_{2*}^{(e)} - \frac{E\alpha_t}{1-\nu}T$$

Retaining only the main terms in the last formula and taking (3.4) into account, we obtain on the edge

$$\tau_2 = \frac{T_2^{(e)}}{2h} + \nu\frac{3\gamma}{2h^3}t - \frac{E\alpha_t}{1-\nu}T\left(h_*^{-2}\right) \quad \left[\tau_2 = \frac{T_2^{(e)}}{2h} - E\alpha_t T\right]\left(h_*^{-2}\right) \tag{4.1}$$

Here and below those transition formulas are written in the square brackets which take a simpler form for a linear law of temperature variation along the thickness. The asymptotic order of the stresses under the assumption that the quantity t is commensurate with unity is indicated in the parentheses to the right of the formula.

Analogous reasoning shows that the stress τ_2 on a hinge-supported edge is determined by means of (4.1), while the transition formulas for the stress τ_{12} become

$$\tau_{12} = \frac{S_{12}^{(h)} + S_{12}^{(e)}}{2h} + \frac{3\gamma}{2h^3}H_{12}^{(e)} \quad \left(h_*^{-3/2-s}\right)$$

The stresses are determined as follows on a rigidly clamped edge $\alpha_1 = \alpha_{10}$

$$\tau_1 = \tau_2 = -\frac{E\alpha_t}{1-\nu}T \quad \left(h_*^{-2}\right)$$

$$\tau_{12} = \frac{S_{12}^{(p)} + S_{12}^{(h)}}{2h} \quad \left(h_*^{-1-2s}\right)$$

Far from the edges, we obtain in a rigid shell

$$\tau_1 = \tau_2 = -\frac{E\alpha_t}{1-\nu}T \quad \left(h_*^{-2}\right), \qquad \tau_{12} = \frac{S_{12}^{(p)} + S_{12}^{(h)}}{2h}$$

The stresses τ_{12} are on the order of h_*^{-1-2s} if all the shell edges are clamped, and $O\left(h_*^{-3/2+s}\right)$ if the shell has a simply-supported or hinge-supported edge.

It is seen from the formulas obtained that the greatest stresses (both at and far from the edge) are determined by the temperature and the known simple edge effect quantities. The stresses in which the forces of the fundamental state of stress entered, are at least $h_*^{-1/2+s}$ times less than the greatest stresses.

If the determination of the principal stresses is satisfied, there is no need to solve the boundary value problem. However, the solution of the boundary value problem is necessary for a more exact calculation of the stresses. It is also needed in case the displacements are of interest.

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ASYMPTOTIC DETERMINATION OF THE FORMATION PROCESS OF NONLINEAR DISTORTION OF ONE-DIMENSIONAL PULSES IN A LAYERED MEDIUM

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Nonlinear effects in the propagation, reflection, and refraction of one-dimensional pulses in a medium consisting of two layers lying on a half-space are considered and analyzed. Properties of layers and of the half-space are different, and stresses are defined by an expansion in powers of strains. The initial pulse of finite duration is specified in the form of boundary condition at the surface of the external layer either for the deformation or for the dislocation rate, and the problem of wave pattern when the initial pulse amplitude tends to zero, i.e. in the case of small nonlinear effects, is solved.

Problem is solved by the method of successive integration of nonhomogeneous linear wave equations, in which the solution of the linear problem is taken as the first approximation and the subsequent approximations are derived by approximating the nonlinear terms with the use of the preceding approximation.

The derived first approximation formulas make possible to solve the inverse problem of acoustic determination of the properties of a medium by the parameters of reflected